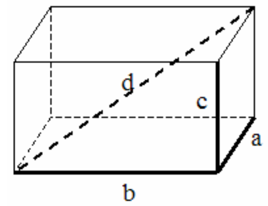
14. GEOMETRIA SOLIDA

Nel seguito: V volume, A_l area laterale, A_b area di base, A_t area totale, $2p_b$ perimetro di base, C circonferenza, d diagonale, h altezza, l lato, r raggio, r_i raggio della sfera inscritta, r_c raggio della sfera circoscritta, a apotema (in alcuni casi può essere un semplice spigolo).

1. Parallelepipedo rettangono



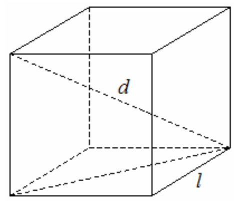
$V = A_b \cdot c = a \cdot b \cdot c \qquad A_l = 2p_b \cdot c \qquad A_b = a \cdot b$

$A_t = 2A_b + A_l = 2(ab + bc + ac) \qquad d = \sqrt{a^2 + b^2 + c^2}$

$A_l = A_t - 2A_b \qquad A_b = \dfrac{A_t - A_l}{2} = \dfrac{V}{c} \qquad 2p_b = \dfrac{A_l}{c}$

Il baricentro è il punto di intersezione delle diagonali.

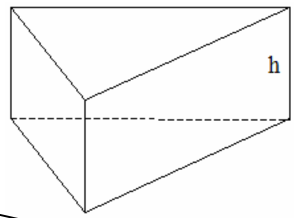
2. Cubo



$V = l^3 \qquad A_l = 4l^2 \qquad A_t = 6l^2 \qquad d = l\sqrt{3}$

$r_i = \dfrac{l}{2} \qquad r_c = \dfrac{l}{2}\sqrt{3} \qquad l = \sqrt[3]{V} = \sqrt{\dfrac{A_t}{6}} = \sqrt{\dfrac{A_l}{4}}$

3. Prisma retto

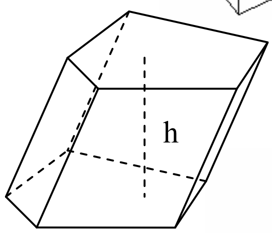


Il prisma retto ha la superficie inferiore congruente e parallela alla superficie superiore, le facce laterali sono rettangoli.

$V = A_b \cdot h \qquad A_l = 2p_b \cdot h \qquad A_t = A_l + 2A_b \qquad 2p_b = \dfrac{A_l}{h}$

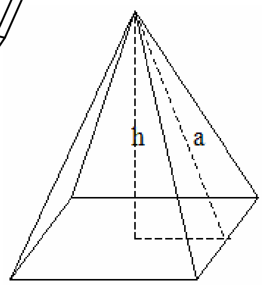
$h = \dfrac{A_l}{2p_b} = \dfrac{V}{A_b} \qquad A_l = A_t - 2A_b \qquad A_b = \dfrac{A_t - A_l}{2} \qquad A_b = \dfrac{V}{h}$

4. Prisma obliquo



$V = A_b \cdot h \qquad A_t = A_l + 2A_b$

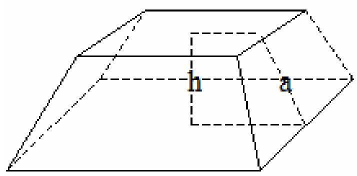
5. Piramide retta



$V = \dfrac{1}{3} A_b \cdot h \qquad A_l = \dfrac{2p_b \cdot a}{2} \qquad A_t = A_b + A_l \qquad A_b = \dfrac{3V}{h}$

$2p_b = \dfrac{2A_l}{a} \qquad a = \dfrac{2A_l}{2p_b} \qquad h = \dfrac{3V}{A_b}$

6. Tronco di piramide



$V = \dfrac{1}{3} \cdot h \cdot (A_b + A_{b'} + \sqrt{A_b \cdot A_{b'}}) \qquad A_l = \dfrac{(2p + 2p') \cdot a}{2}$

$a = \dfrac{2A_l}{2p + 2p'} \qquad A_t = A_l + A_b + A_{b'}$

7. Poliedri regolari

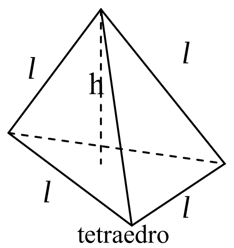
Area e volume si possono calcolare in maniera approssimata utilizzando i numeri fissi φ e σ

$A=\varphi\cdot l^2 \qquad V=\sigma\cdot l^3$

Poliedro	Tetraedro	Esaedro o cubo	Ottaedro	Dodecaedro	Icosaedro
Numero fisso per l'area φ	1,73	6	3,464	20,64	8,66
Numero fisso per il volume σ	0,118	1	0,471	7,663	2,182

Tetraedro: formato da 4 triangoli equilateri

$V=\frac{l^3\sqrt{2}}{12} \qquad A_t=l^2\sqrt{3} \qquad r_i=\frac{l\sqrt{6}}{12} \qquad r_c=\frac{l\sqrt{6}}{4}$



tetraedro

Esaedro: formato da 6 quadrati è il cubo

Ottaedro: formato da 8 triangoli equilateri

$V=\frac{l^3\sqrt{2}}{3} \qquad A_t=2l^2\sqrt{3} \qquad r_i=\frac{l\sqrt{6}}{6} \qquad r_c=\frac{l\sqrt{2}}{2}$

Dodecaedro: formato da 12 pentagoni regolari

$V=\frac{l^3\left(15+7\sqrt{5}\right)}{4} \qquad A_t=3l^2\sqrt{5\left(5+2\sqrt{5}\right)} \qquad r_i=\frac{l\sqrt{10\left(25+11\sqrt{5}\right)}}{20} \qquad r_c=\frac{l\sqrt{3}\left(1+\sqrt{5}\right)}{4}$

Icosaedro: formato da 20 triangoli equilateri

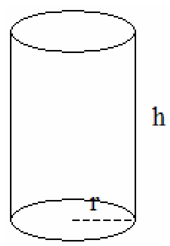
$V=\frac{5l^3\left(3+\sqrt{5}\right)}{12} \qquad A_t=5l^2\sqrt{3} \qquad r_i=\frac{l\sqrt{3}\left(3+\sqrt{5}\right)}{12} \qquad r_c=\frac{l}{4}\sqrt{2\left(5+\sqrt{5}\right)}$

8. Cilindro

$V=A_b\cdot h=\pi r^2 h \qquad A_b=\pi r^2 \qquad A_l=C\cdot h=2\pi r h$

$A_t=A_l+2A_b=2\pi r(h+r) \qquad A_b=\frac{V}{h}$

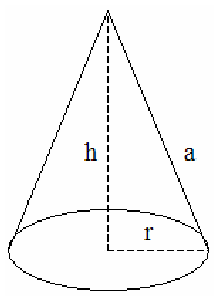
$C=2\pi r=\frac{A_l}{h} \qquad h=\frac{A_l}{2\pi r}=\frac{V}{\pi r^2} \qquad r=\frac{A_l}{2\pi h}=\sqrt{\frac{V}{\pi h}}$



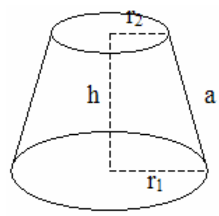
9. Cono

$V=\frac{A_b\cdot h}{3}=\frac{\pi\cdot r^2\cdot h}{3} \qquad A_l=\frac{C\cdot a}{2}=\pi r a \qquad A_b=\pi r^2$

$A_t=A_b+A_l=\pi r^2+\pi r a \qquad a=\frac{A_l}{\pi r} \qquad r=\frac{A_l}{\pi a}=\sqrt{\frac{3V}{\pi h}} \qquad h=\frac{3\cdot V}{\pi r^2}$



10. Tronco di cono



$$V = \frac{1}{3} h\pi\left(r_1^2 + r_1 r_2 + r_2^2\right) \qquad A_l = \pi \cdot a \cdot (r_1 + r_2) \qquad A_b = \pi r_1^2 + \pi r_2^2$$

$$a = \sqrt{h^2 + (r_1 - r_2)^2}$$

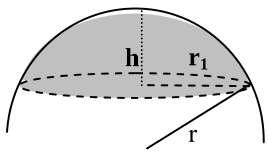
11. Sfera

$$V = \frac{4}{3}\pi r^3 \qquad A = 4\pi r^2 \qquad r = \sqrt{\frac{A}{4\pi}} = \sqrt[3]{\frac{3V}{4\pi}}$$

Calotta sferica e segmento sferico ad una base o sezione sferica

$$V = \frac{1}{3}\pi h^2 (3r - h) \qquad A = 2\pi r h$$

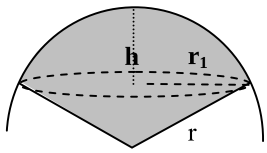
$$r_1 = \sqrt{h(2r - h)}$$



Settore sferico

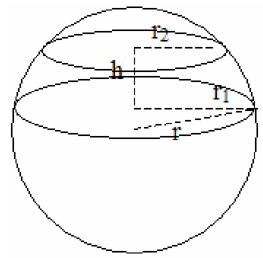
$$A_l = \pi r\left(r_1 + 2h\right)$$

$$V = \frac{2}{3}\pi r^2 h$$



Zona sferica e segmento sferico a due basi

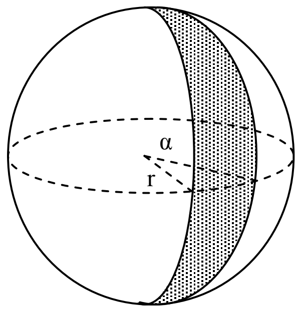
$$V = \frac{\pi \cdot h}{2} \cdot \left(\frac{h^2}{3} + r_1^2 + r_2^2\right) \qquad A = 2\pi r h$$



Fuso sferico e spicchio sferico

$$V = \frac{\pi r^3}{270°}\alpha \qquad A_l = \frac{\pi r^2}{90°}\alpha,$$

α è misurato in gradi
A_l è la parte di superficie sferica

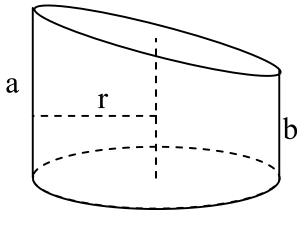


12. Altre figure particolari

Cilidro circolare retto a sezione obliqua

$$V = \pi r^2 \frac{(a+b)}{2} \qquad A_l = \pi r\left(a + b\right)$$

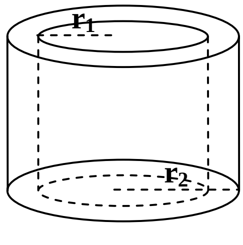
$$A_t = \pi r\left(a + b + r + \sqrt{r^2 + \left(\frac{a-b}{2}\right)^2}\right)$$



Corona cilindrica

$$V = \pi h\left(r_1^2 - r_2^2\right) \qquad A_l = 2\pi h\left(r_1 + r_2\right)$$

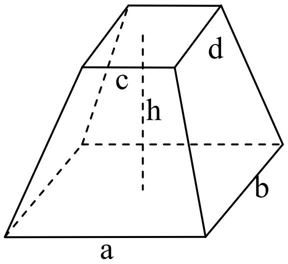
$$A_t = 2\pi\left(r_1 + r_2\right)\left(h + r_1 - r_2\right)$$



Obelisco

Le superfici laterali sono trapezi, le superfici superiore e inferiore sono rettangoli non simili.

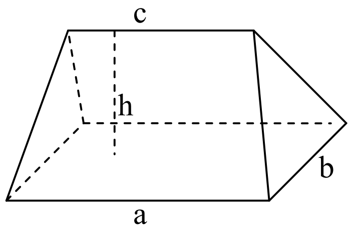
$$V = \frac{h}{6}\left[\left(2a+c\right)b+\left(2c+a\right)d\right]$$



Cuneo

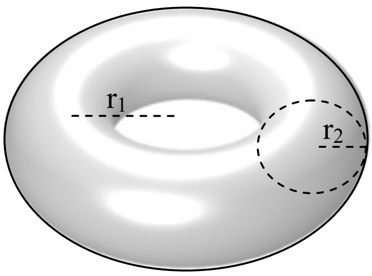
Superficie di base rettangolare, le superfici laterali sono triangoli e trapezi isosceli.

$$V = \frac{bh}{6}\left(2a+c\right)$$



Toro

$$V = 2\pi^2 r_2^2 r_1 \qquad A_t = 4\pi^2 r_1 r_2$$



Prisma obliquo triangolare

$$V = A_b \frac{a+b+c}{3}$$